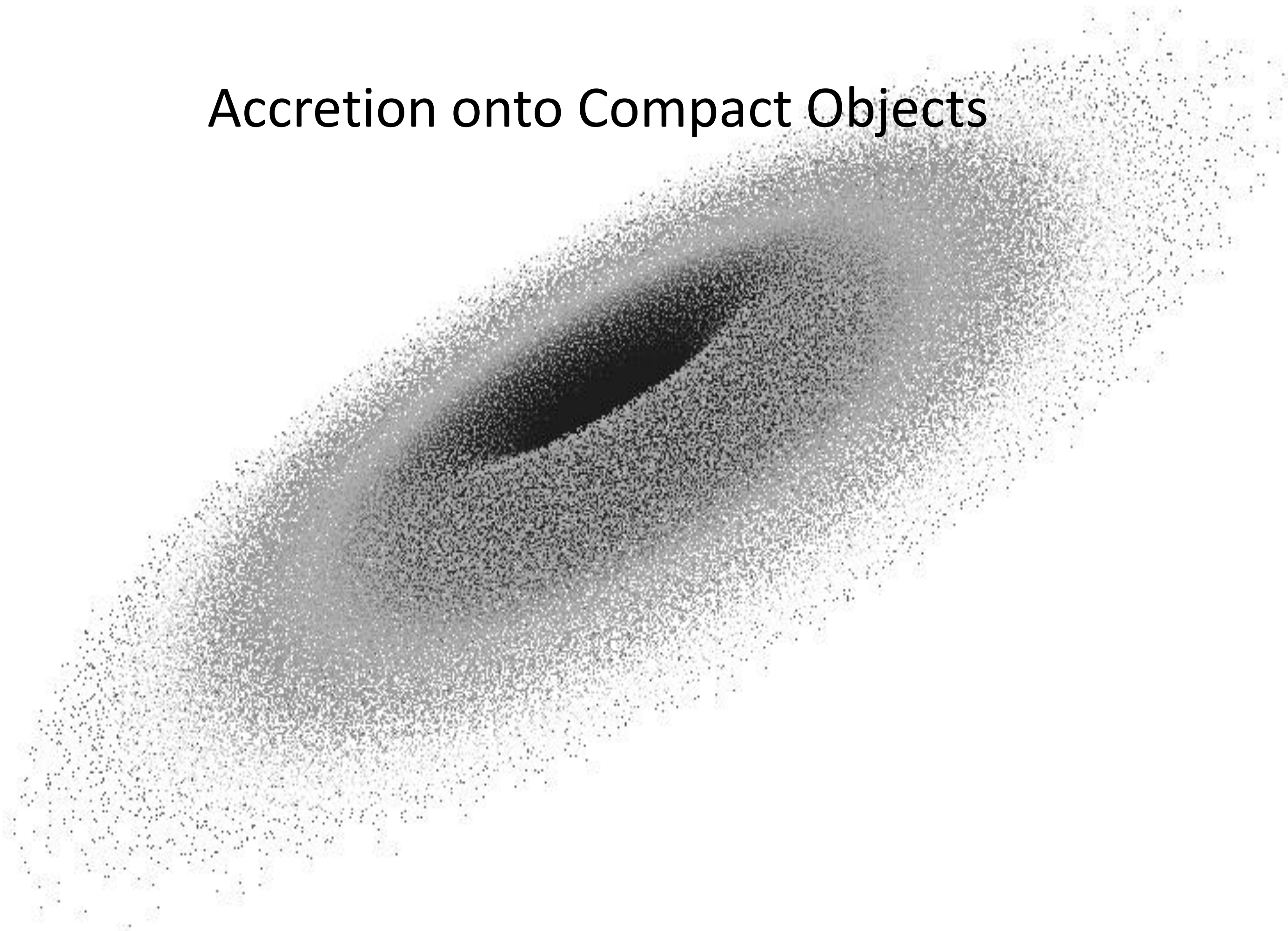
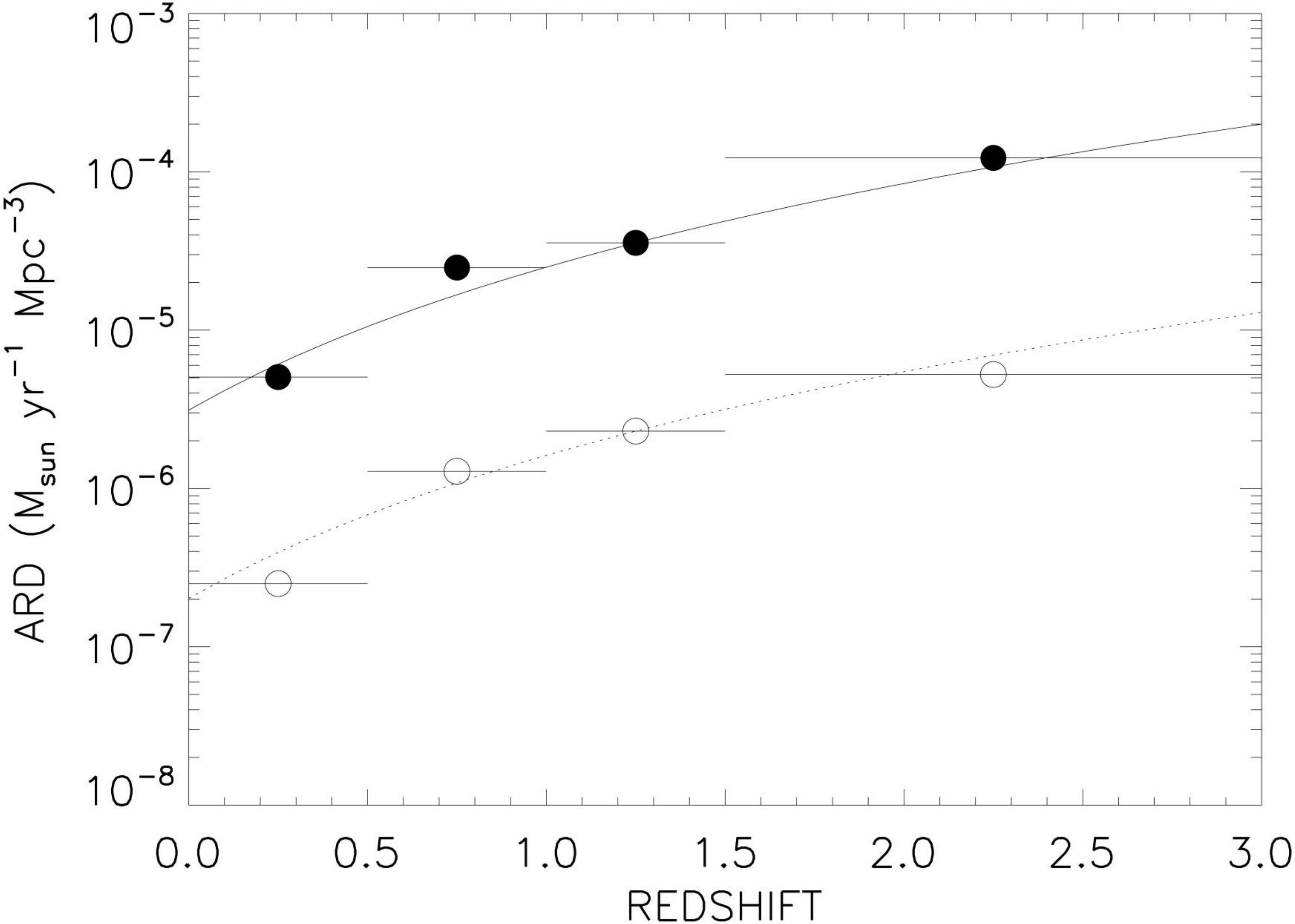
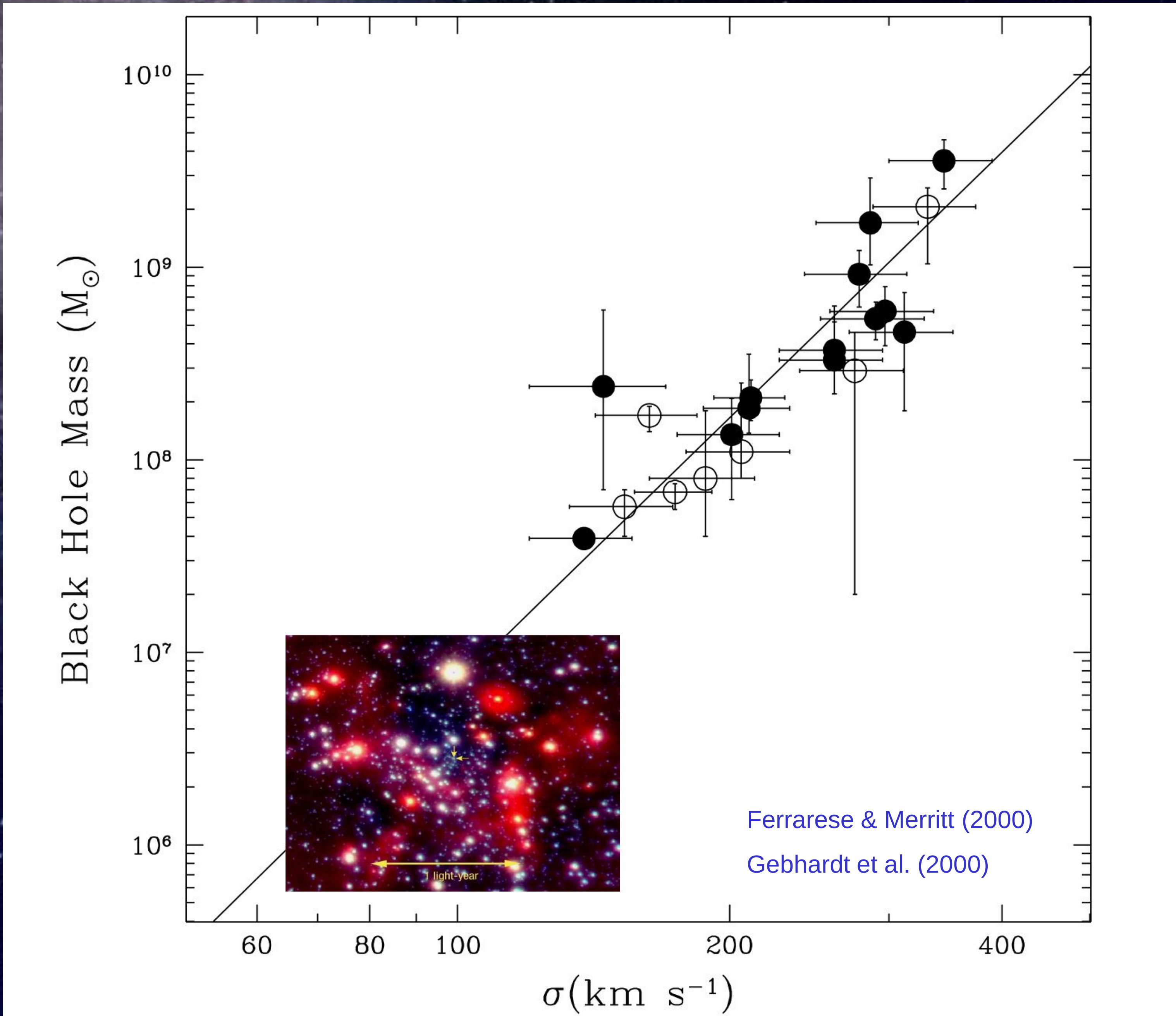
Accretion onto Compact Objects

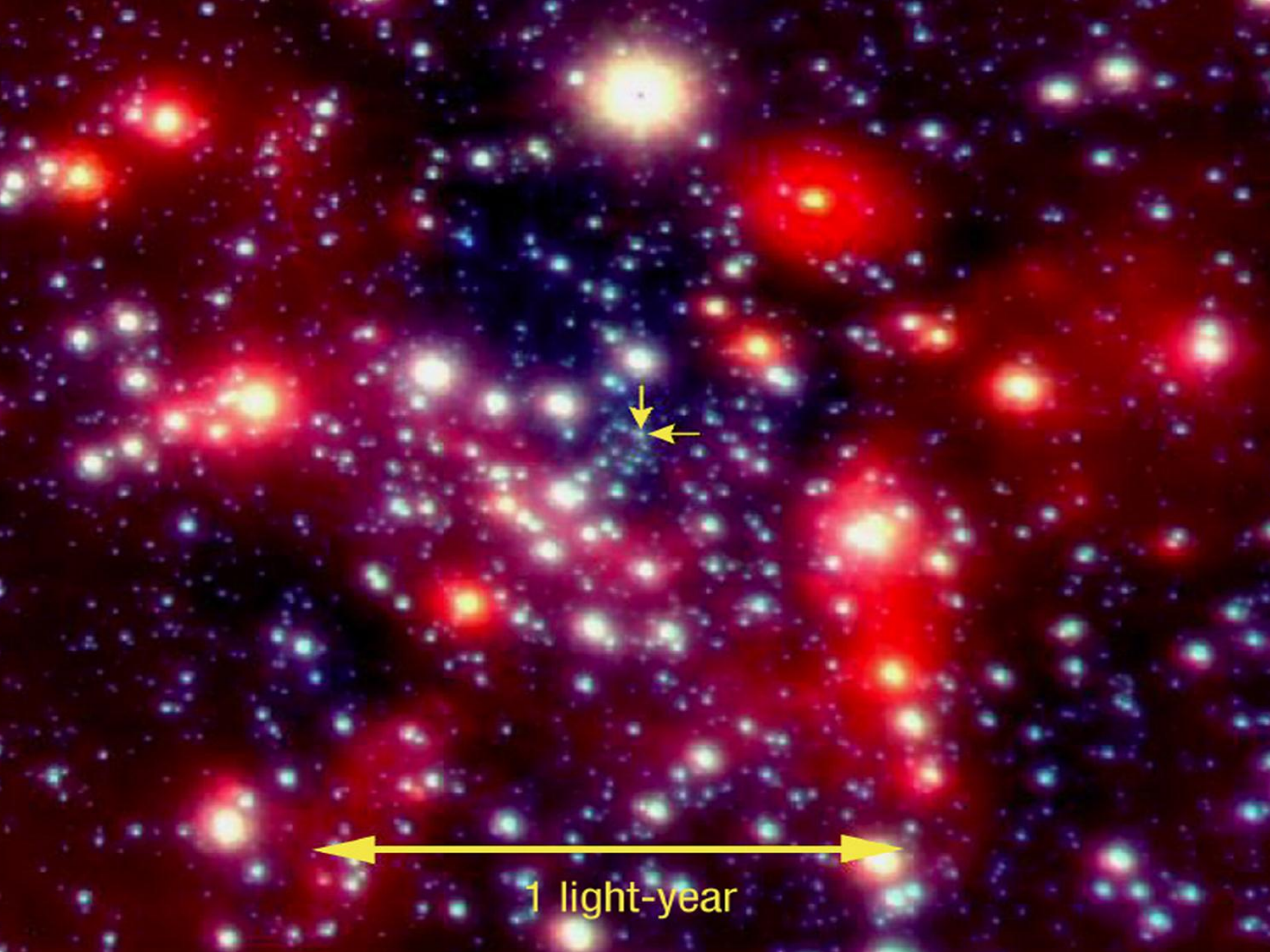


(CDFN: Barger et al. 2002; See also Giacconi et al. 2002 for CDFS)



ARD (M_{sun} yr^{-1} Mpc^{-3})
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
REDSHIFT


Black Hole Mass ($M_\odot$)
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
60
80
100
200
400
$\sigma(\mathrm{km\ s^{-1}})$
1 light-year
Ferrarese & Merritt (2000)
Gebhardt et al. (2000)


1 light-year

6 September 2001

International weekly journal of science

nature

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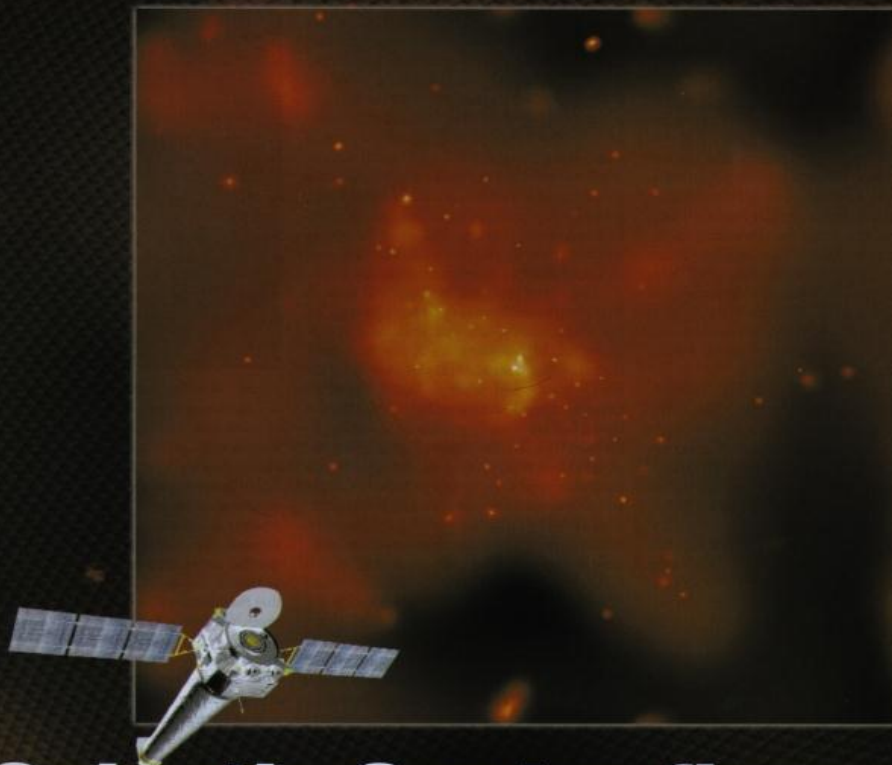


Galactic Centre flares up

Bacterial cytoskeleton Actin's ancestral ribbons

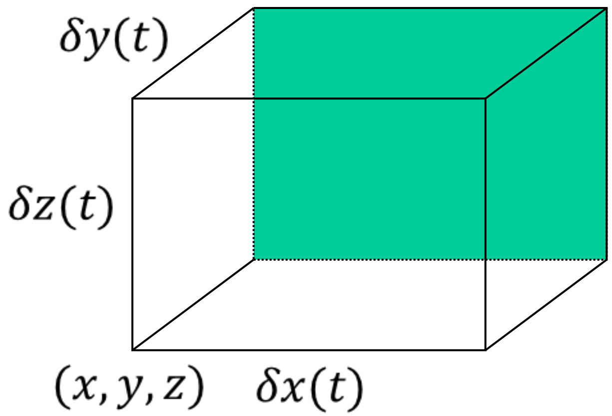
Earth's inner core Anisotropy explained?

Cancer Dual role of $p16^{Ink4a}$ and $p19^{Arf}$

nature jobs
structural biology

Classical Hydrodynamics

Mass Conservation:



$$\delta V(t) = \delta x(t)\delta y(t)\delta z(t)$$

Conservation of mass requires

$$\frac{dm}{dt} = 0$$

$$\frac{d(\rho\delta V)}{dt} = \delta V\frac{d\rho}{dt} + \rho\frac{d(\delta V)}{dt} = 0$$

$$\begin{aligned}
d(\delta V) = &\left[-v_x\delta t\delta y\delta z + \left(v_x + \frac{\partial v_x}{\partial x}dx\right)dt\delta y\delta z\right] + \\
&\left[-v_y\delta t\delta x\delta z + \left(v_y + \frac{\partial v_y}{\partial y}dy\right)dt\delta x\delta z\right] + \\
&\left[-v_z\delta t\delta x\delta y + \left(v_z + \frac{\partial v_z}{\partial z}dz\right)dt\delta x\delta y\right]
\end{aligned}$$

Conservation Equations

$$\frac{1}{\delta V}\frac{d(\delta V)}{dt} = \vec{\nabla}\cdot\vec{v}$$

$$\delta V\frac{d\rho}{dt} + \rho\frac{d(\delta V)}{dt} = 0 \longrightarrow \frac{d\rho}{dt} + \rho\left(\vec{\nabla}\cdot\vec{v}\right) = 0$$

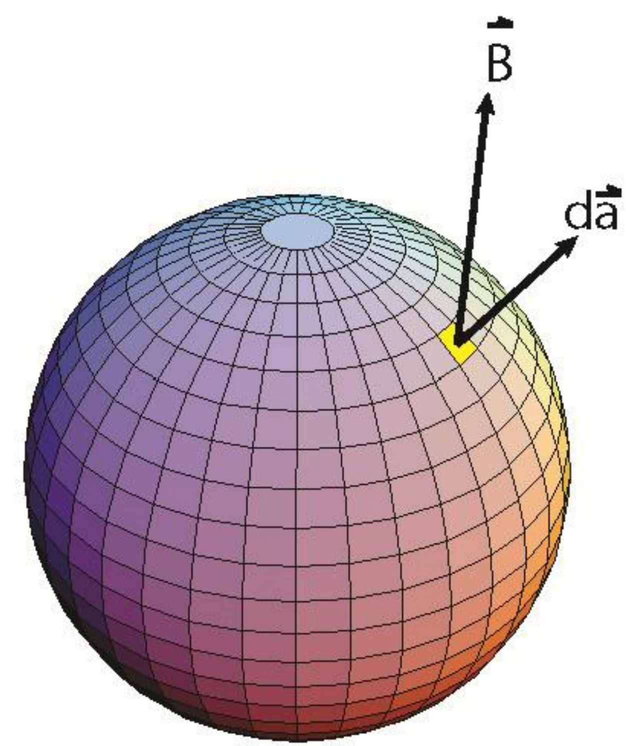
Or using the advective time derivative, $\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$

$$\frac{\partial\rho}{\partial t} + \vec{v}\cdot\vec{\nabla}\rho + \rho\left(\vec{\nabla}\cdot\vec{v}\right) = 0$$

$$\boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$$

(Conservation of mass)

$$\nabla \cdot \vec{B} = \lim_{V \to 0} \frac{1}{V} \oint_S \vec{B} \cdot d\vec{a}$$



Conservation Equations

Conservation of momenutm

$$\frac{d(\rho\vec{v})}{dt} + \rho\vec{v}\left(\vec{\nabla}\cdot\vec{v}\right) = -\vec{\nabla}\cdot\tilde{\sigma} + \frac{1}{\delta V}\vec{F}_b$$

$\sigma_{ij} =$ *i*-momentum density flux across the *j*-surface

$-\vec{\nabla}\cdot\tilde{\sigma} = -\vec{\nabla}P$ (when stress is isotropic)

$\vec{F}_b =$ body force (e.g., gravity)

Or using the advective derivative again,

$$\boxed{\frac{\partial(\rho\vec{v})}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}\vec{v}) = -\vec{\nabla}\cdot\tilde{\sigma} + \frac{1}{\delta V}\vec{F}_b}$$

(Conservation of momentum)

Conservation Equations

In similar fashion,

$$\boxed{\frac{\partial\left(\frac{1}{2}\rho v^2\right)}{\partial t} + \vec{\nabla}\cdot\left(\frac{1}{2}\rho v^2\vec{v}\right) = -\vec{v}\cdot\left(\vec{\nabla}\cdot\tilde{\sigma}\right) + \vec{v}\cdot\frac{\vec{F}_b}{\delta V}}$$

(Conservation of kinetic energy density)

and

$$\boxed{\frac{\partial(\rho u)}{\partial t} + \vec{\nabla}\cdot(\rho u\vec{v}) = \rho T\frac{ds}{dt} - P\left(\vec{\nabla}\cdot\vec{v}\right)}$$

(Conservation of internal energy density)

Conservation Equations

Combined energy equation: put $\tilde{\sigma} = P\tilde{I}$ (isotropic pressure)

$$\boxed{\frac{\partial\left(\frac{1}{2}\rho v^2 + \rho u\right)}{\partial t} + \vec{\nabla}\cdot\left[\rho\vec{v}\left(\frac{1}{2}v^2 + u + \frac{P}{\rho}\right)\right] = \vec{v}\cdot\frac{\vec{F}_b}{\delta V} - \vec{\nabla}\cdot\vec{F}_{rad} - \vec{\nabla}\cdot\vec{q}}$$

where $u =$ internal energy per unit mass

$u + \frac{P}{\rho} =$ enthalpy per unit mass

$\vec{F}_{rad} =$ radiative flux vector

$\vec{q} =$ conductive heat flux

Radial Infall

For zero angular momentum (in steady state):

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho v) = 0$$

$$\boxed{4\pi r^2\rho(-v) = \dot{M}}$$ (mass)

$$\boxed{v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM}{r^2} = 0}$$ (momentum)

$$\boxed{P = K\rho^\gamma}$$ (polytrope)

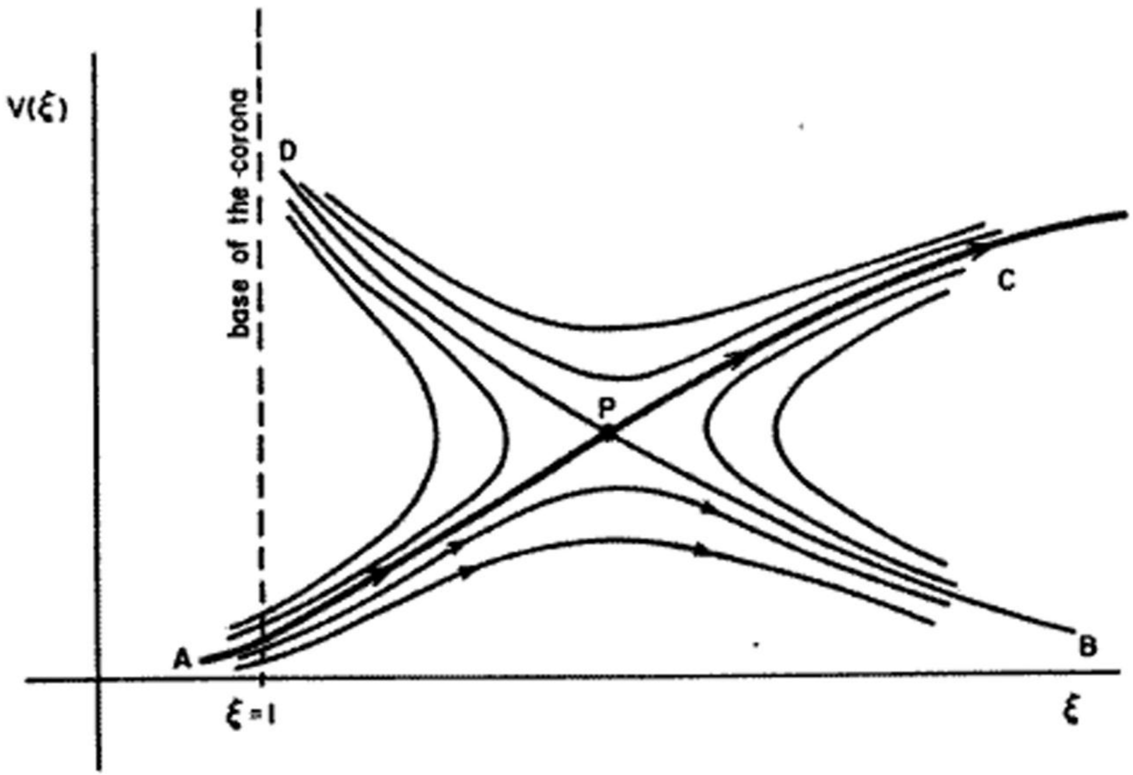
Adiabatic: $\gamma = 5/3$ Isothermal: $\gamma = 1$

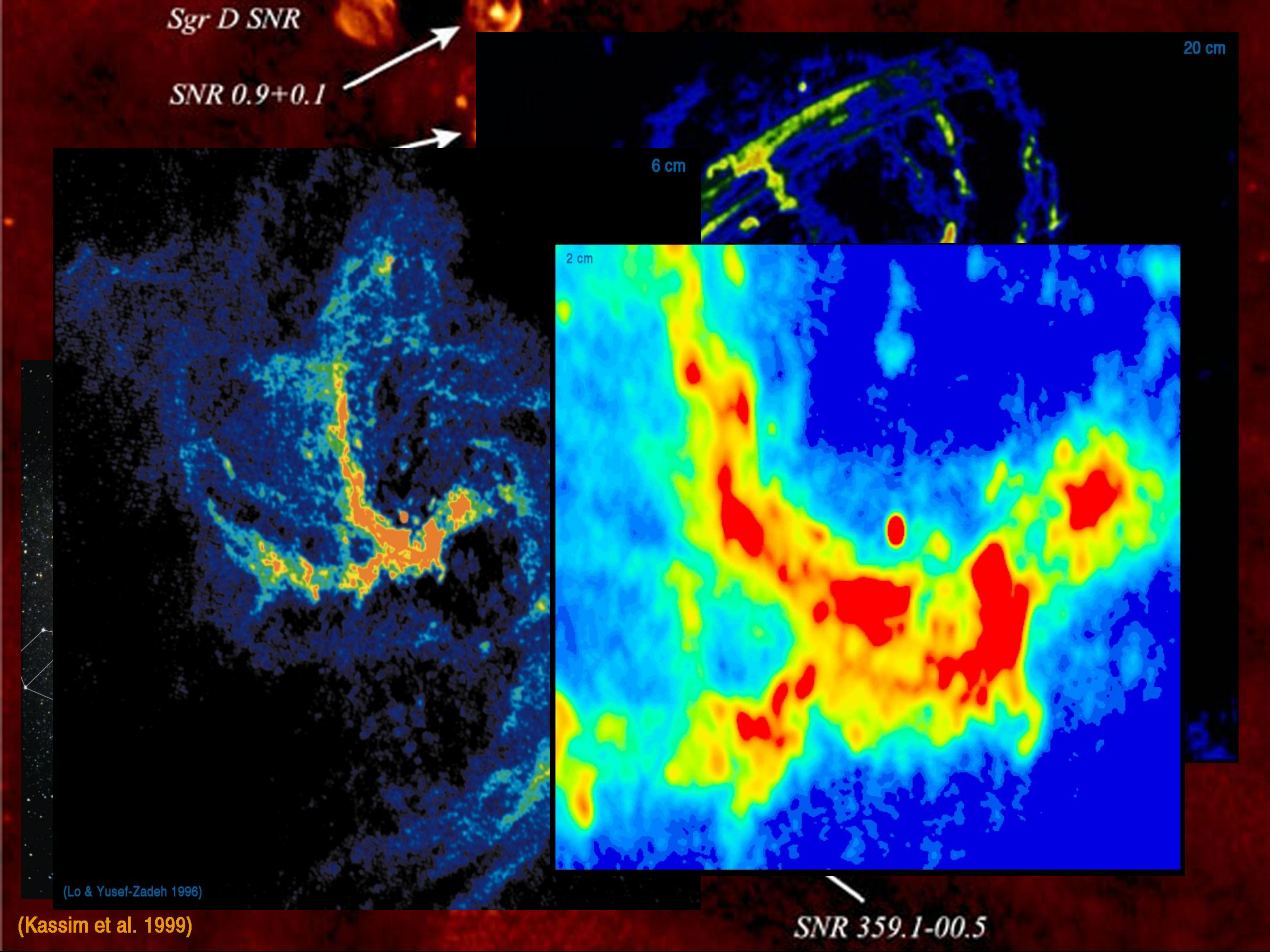
Analytic Solution

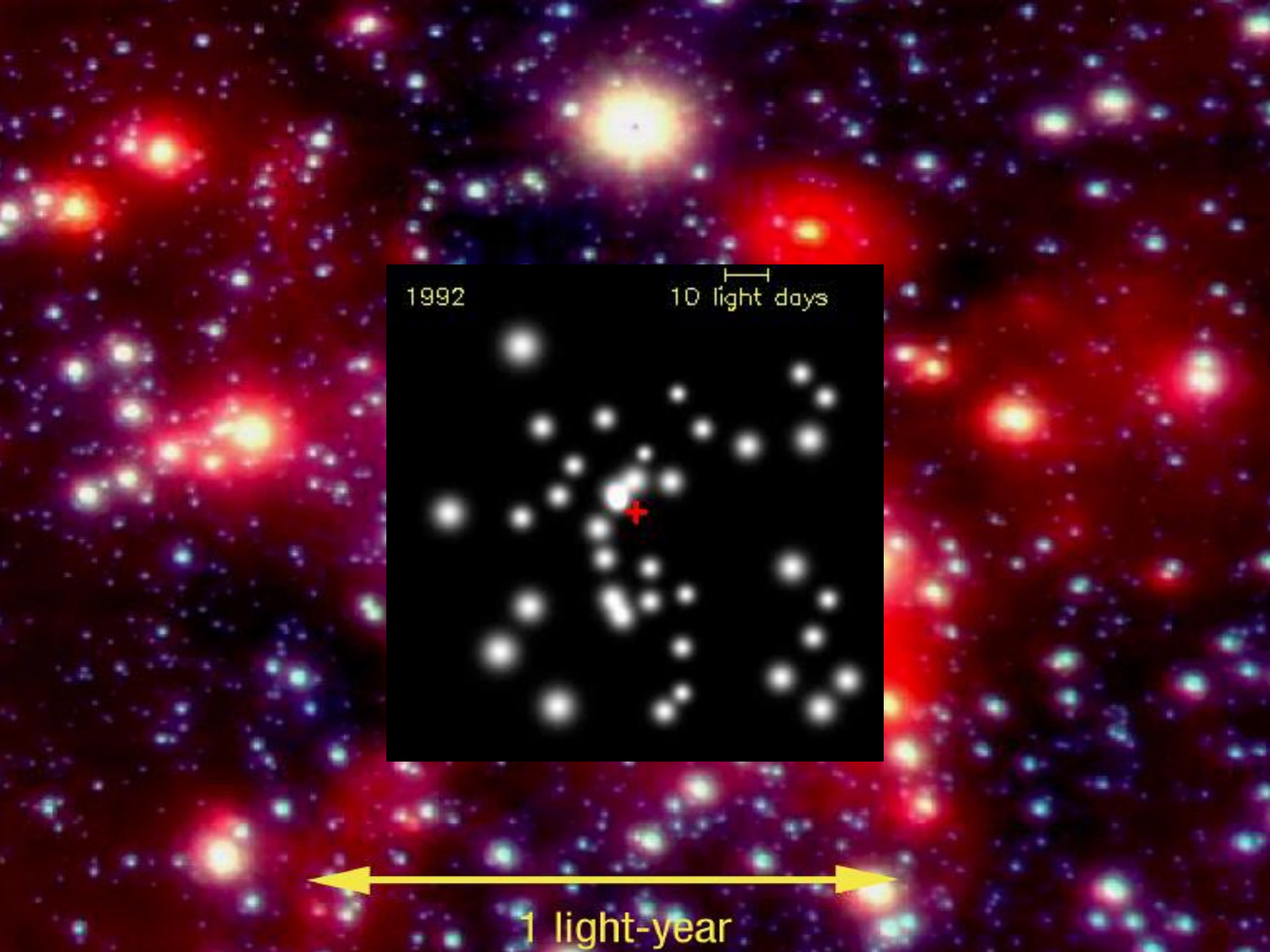
Combining the mass and momentum equations:

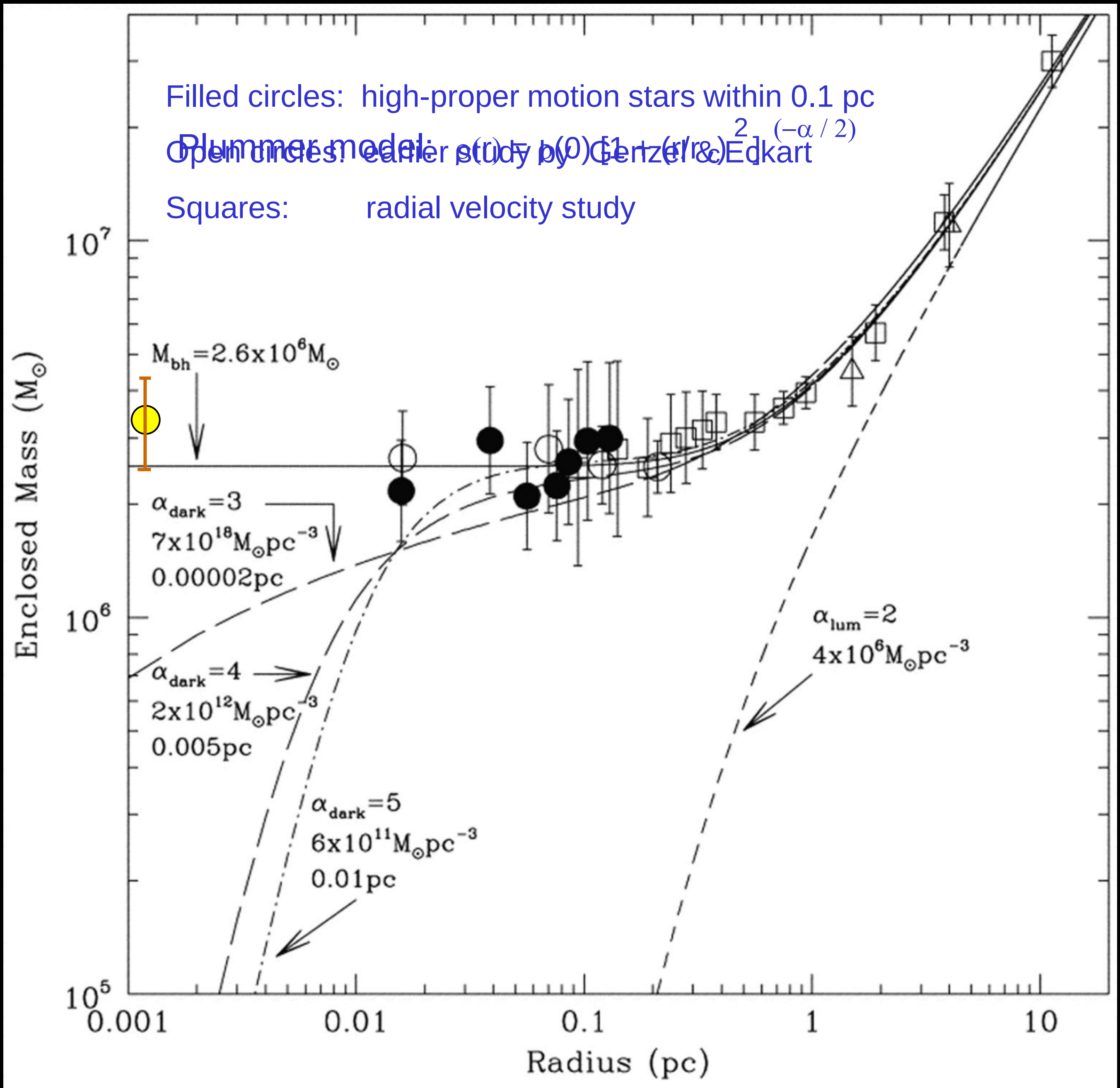
$$\frac{1}{2}\left(1-\frac{{c_s}^2}{v^2}\right)\frac{d}{dr}(v^2) = \frac{GM}{r^2}\left(1-\frac{2{c_s}^2}{GM}\right)$$

where $c_s = \left(\frac{dP}{d\rho}\right)^{1/2}$ is the sound speed

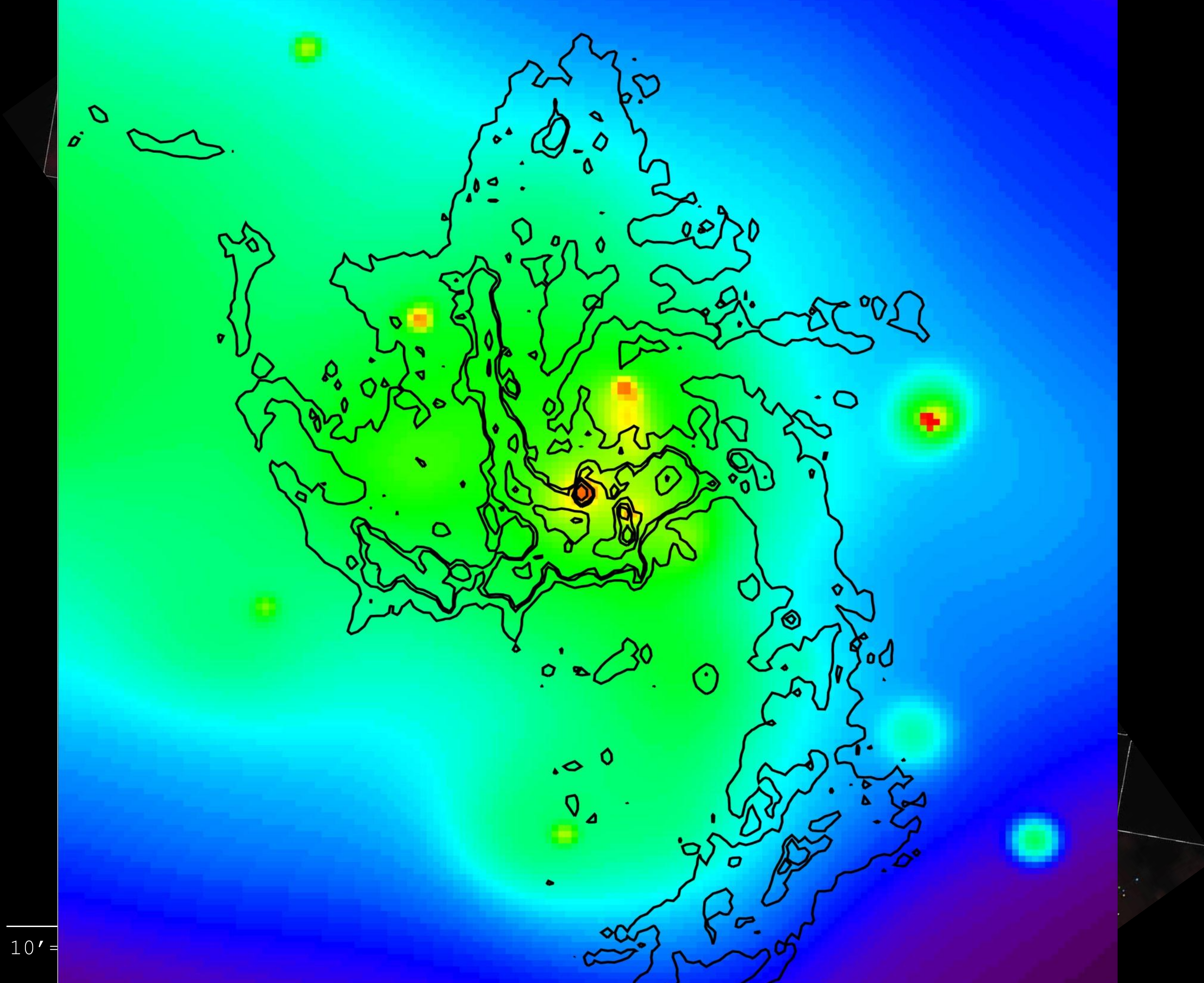


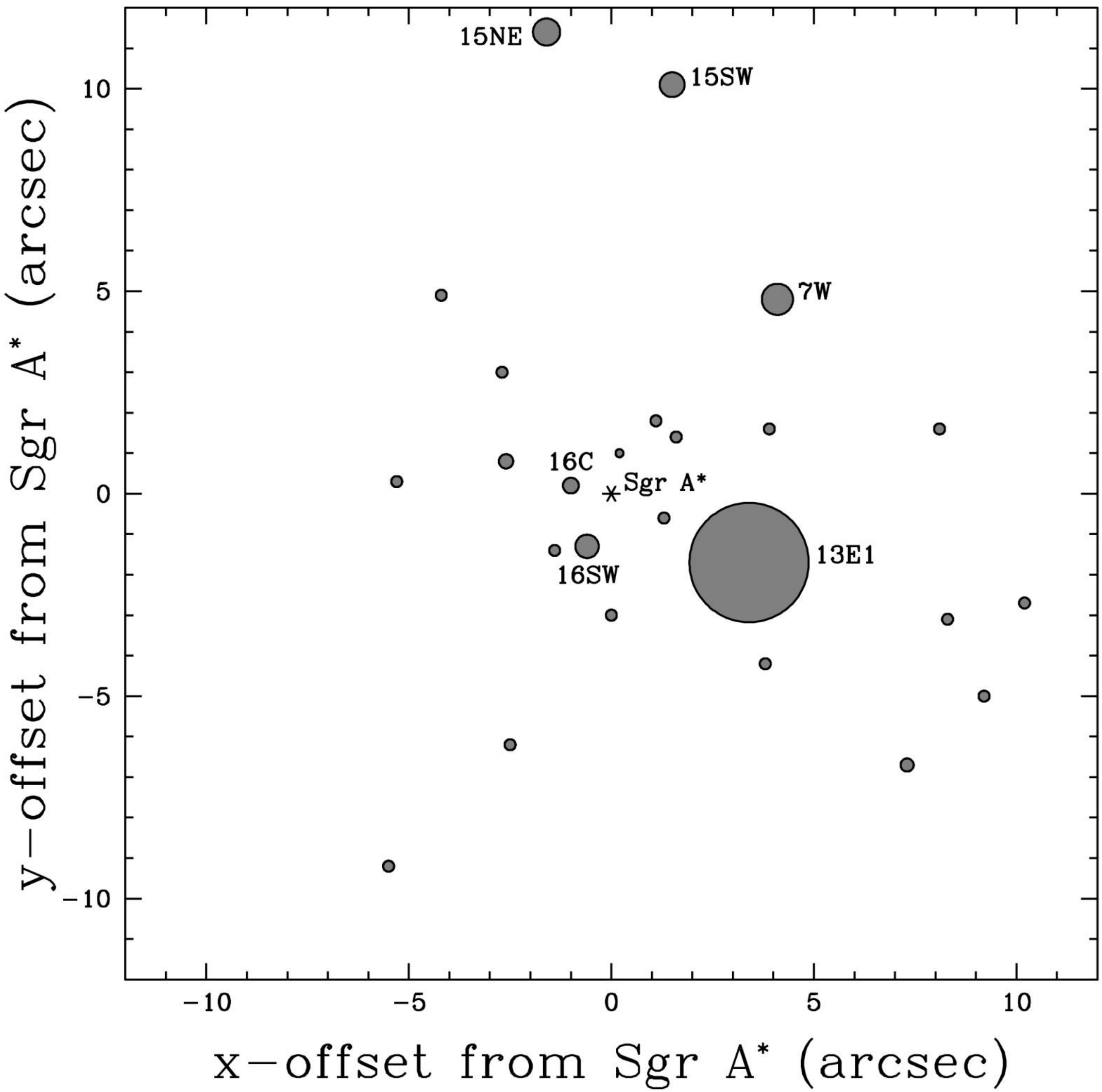

Sgr D SNR
SNR 0.9+0.1
20 cm
6 cm
2 cm
(Lo & Yusef-Zadeh 1996)
(Kassim et al. 1999)
SNR 359.1-00.5

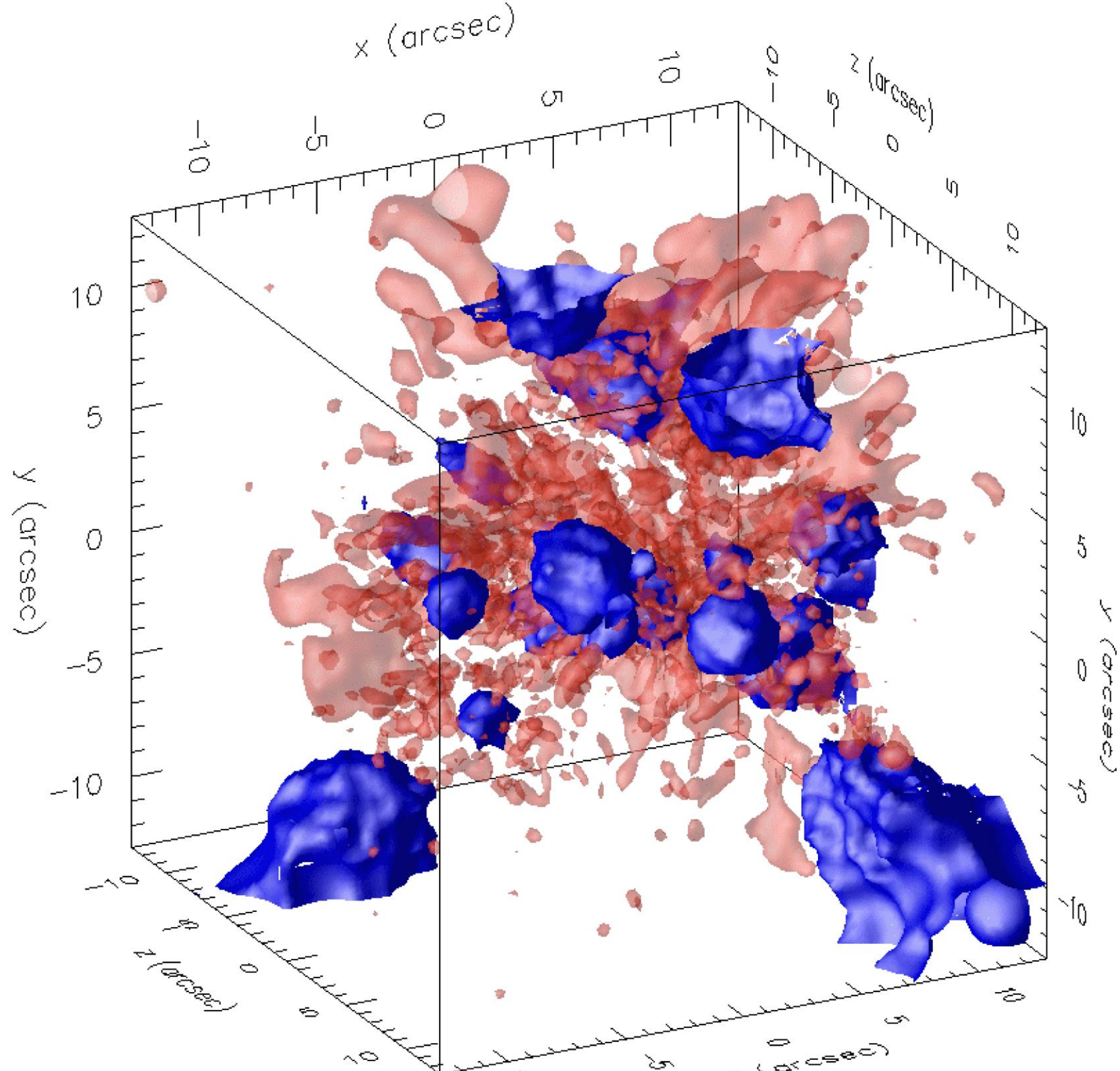

1992
10 light days
1 light-year



Filled circles: high-proper motion stars within 0.1 pc
Plummer model: ρ(r) = ρ(0)[1 + (r/rc)2](−α / 2)
Open circles: earlier study by Genzel & Eckart
Squares: radial velocity study
Mbh=2.6x10^6M⊙
αdark=3
7x10^18M⊙pc^-3
0.00002pc
αdark=4
2x10^12M⊙pc^-3
0.005pc
αdark=5
6x10^11M⊙pc^-3
0.01pc
αlum=2
4x10^6M⊙pc^-3
Enclosed Mass (M⊙)
10^7
10^6
10^5
0.001
0.01
0.1
1
10
Radius (pc)


10′=

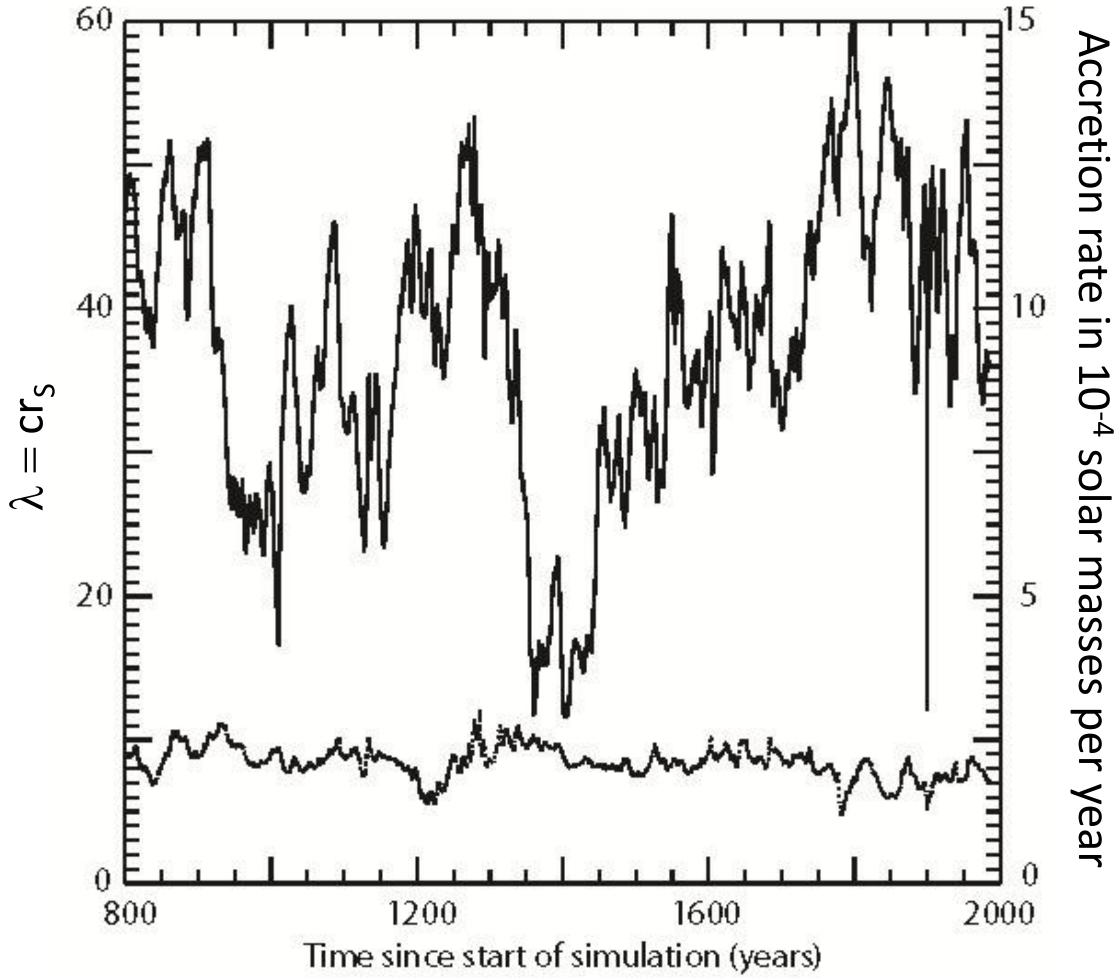

15NE
15SW
7W
16C
Sgr A*
13E1
16SW
10
5
0
−5
−10
y−offset from Sgr A* (arcsec)
x−offset from Sgr A* (arcsec)


x (arcsec)
z (arcsec)
y (arcsec)
y (arcsec)
z (arcsec)
x (arcsec)


t = 0.349
VIDEO CALIBRATION

The mass accretion rate shows ~ 100 year variability with 20 - 40% fluctuations.

The specific angular momentum varies both in magnitude and sign on a similar time scale. So if a **small** disk forms, it can dissolve and reform every few hundred years.



Magnetohydrodynamics

Introducing a magnetic field B creates 3 new effects: (1) currents, (2) a Lorentz force (that can change the momentum of the plasma), and (3) an evolution (growth or decay) in B with a subsequent energy exchange with the plasma.

(1) $$\vec{J}' = \sigma \vec{E}'$$ (Ohm's law)

$$\vec{E}' = \vec{E} + \frac{\vec{v}}{c} \times \vec{B}$$

$$\boxed{\vec{E}' = \vec{J}/\sigma \approx 0}$$ (for most astrophysical applications, $\sigma \to \infty$)

$$\boxed{\vec{E} \approx -\frac{\vec{v}}{c} \times \vec{B}}$$ (the IDEAL MHD condition)

(2) $$\frac{\partial(\rho\vec{v})}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}\vec{v}) = -\nabla P + \vec{f}_b \boxed{+ \frac{1}{c}(\vec{J}\times\vec{B})}$$

$$\boxed{\frac{\partial(\rho\vec{v})}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}\vec{v}) = -\nabla P + \vec{f}_b - \frac{1}{4\pi}\vec{B}\times(\vec{\nabla}\times\vec{B})}$$

(3) From Faraday's law

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \vec{E}$$

$$\vec{E} \approx -\frac{\vec{v}}{c} \times \vec{B}$$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right)}$$

$$\boxed{\frac{\partial (\rho\vec{v})}{\partial t} + \vec{\nabla} \cdot (\rho\vec{v}\vec{v}) = -\nabla P + \vec{f}_b - \frac{1}{4\pi}\vec{B} \times \left(\vec{\nabla} \times \vec{B}\right)}$$

First Generation MHD Simulations

Assumptions: (1) Weakly resistive MHD, in which $\vec{E} \approx -(\vec{v}/c) \times \vec{B}$, but with η (resistivity) not zero.

(2) Ideal gas (no radiation), for which $P = (\gamma - 1)\rho\epsilon$ (with $\gamma = 5/3$) and no radiative cooling (i.e., a low efficiency radiator)

(3) $M = 1.7$ (Mach number)

(4) 3D MHD with radial boundary conditions

(5) Weak initial (uniform) magnetic field: $\vec{B}_0 = B_0\hat{z}$

Igumenshchev et al. (2001)

What happens physically?

If B is frozen into the ionized plasma, then $B \propto r^{-2}$ due to flux conservation.

So $u_B \equiv B^2/8\pi \propto r^{-4}$. But $u_G \equiv GM\rho/r \propto r^{-5/2}$ (for gas in free-fall).

So eventually at small enough r, the magnetic field becomes super-equipartition.

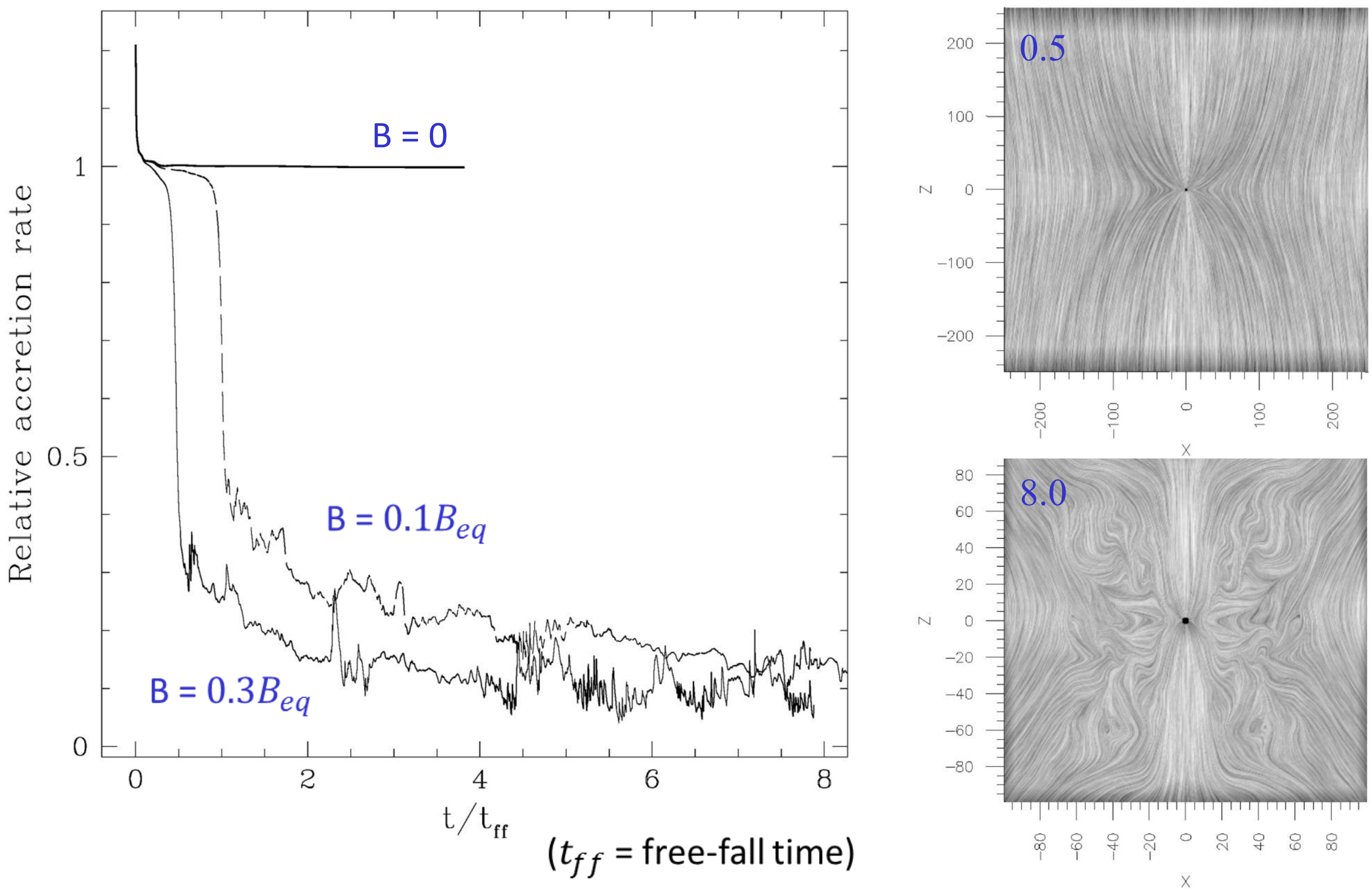
Assume that the excess B reconnects and goes directly into heat:

$$\boxed{\left(B^2 - {B_{eq}}^2\right)/8\pi \rightarrow nk\Delta T}$$

This heats the gas substantially, especially at small radii, and leads to

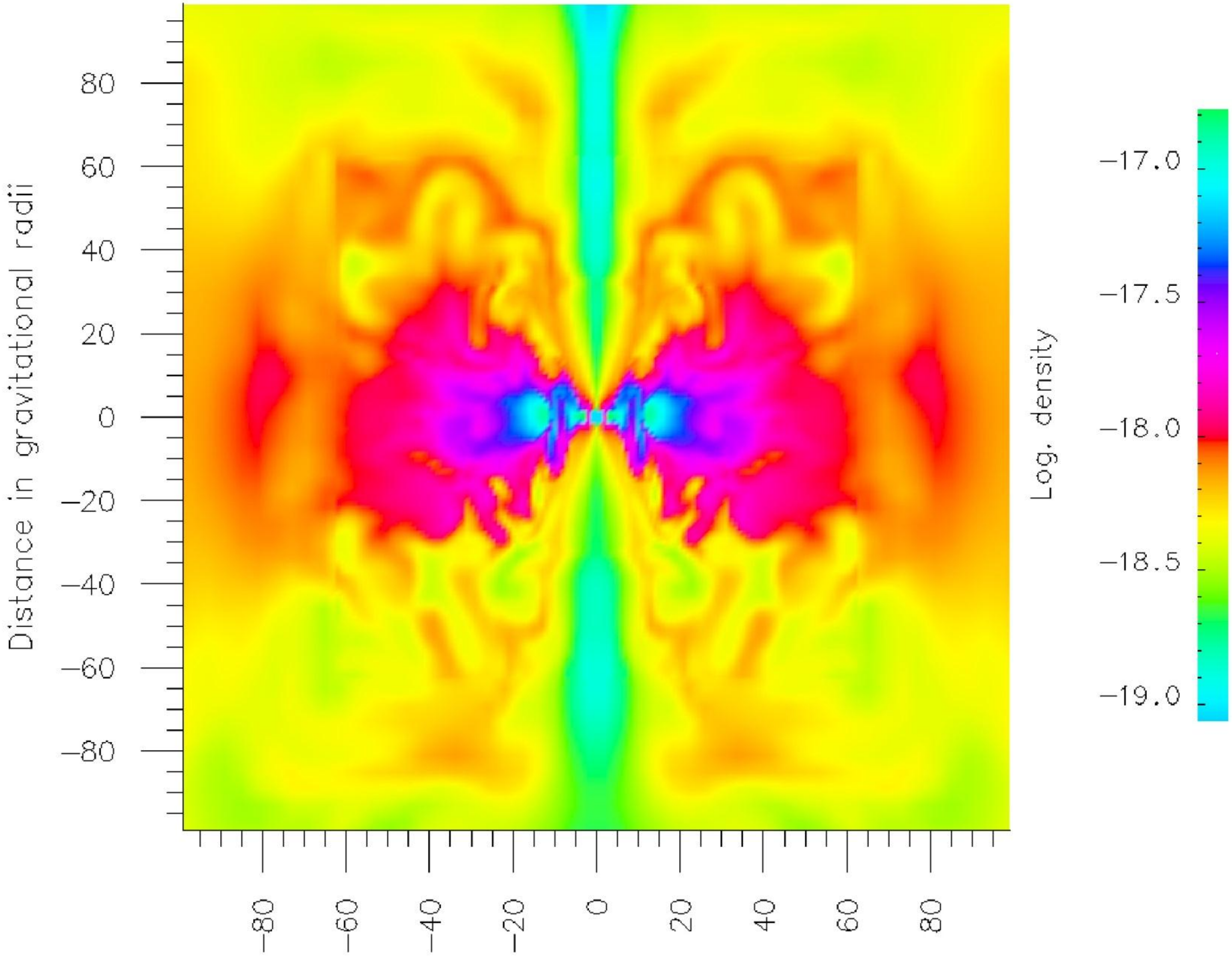
self-sustained convection. The velocity is greatly subsonic everywhere.

Some results

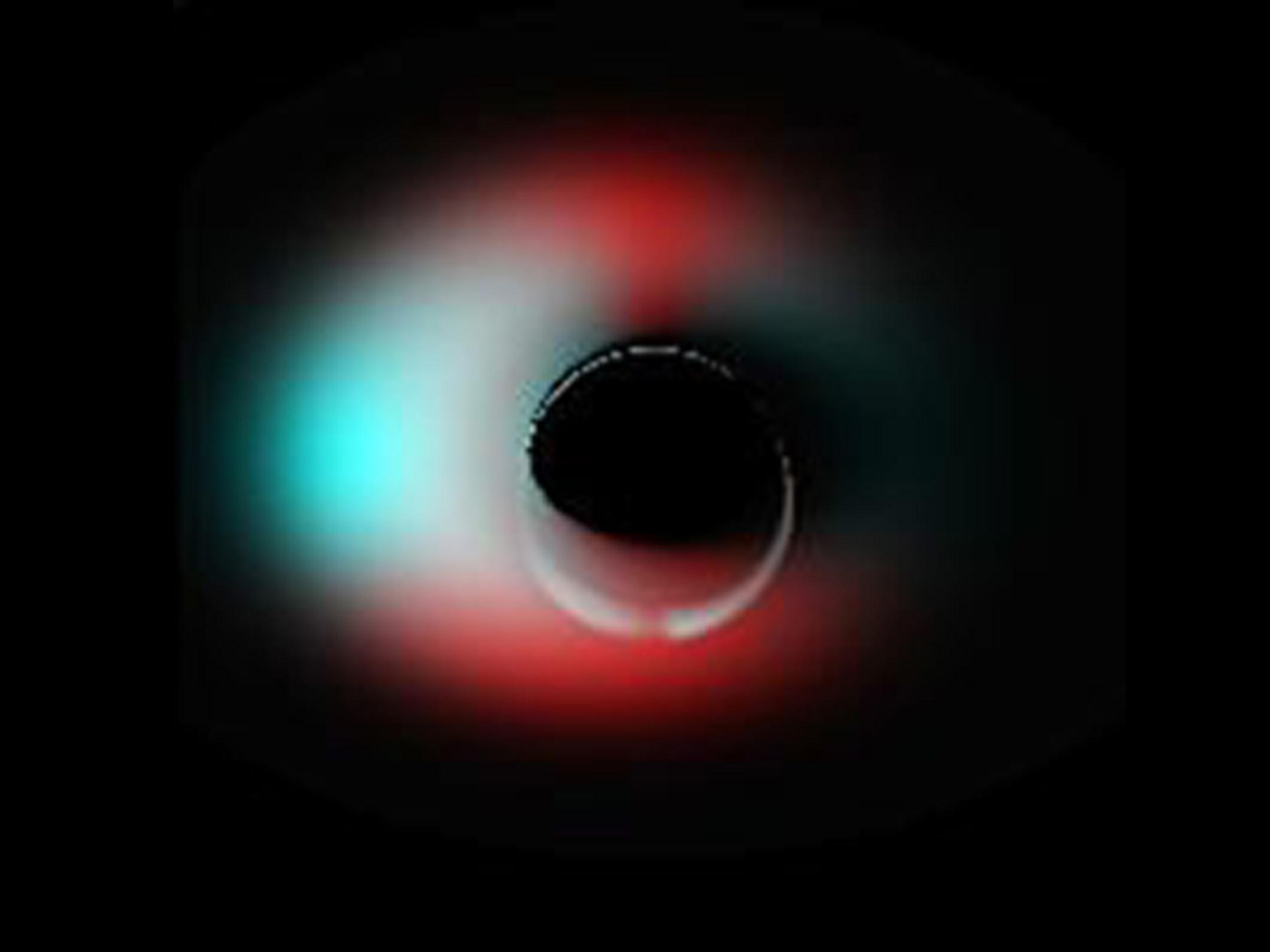


(t_{ff} = free-fall time)

Density Map after 8 free-fall times



(Igumenshchev et al. 2001)



Instead of $\rho \propto r^{-3/2}$ as in the free-fall case, here $\rho \propto r^{-1/2}$.

The net result is that the rate of accretion drops everywhere compared to the pure Bondi-Hoyle value:

$$\boxed{\dot{M} \approx \left(\frac{r_{in}}{r_{cap}}\right) \dot{M}_{BH}}$$

$r_{in} \approx \delta r_S$ (δ typically < 10)

So if the capture radius $r_{cap} \sim 10^5$, the accretion rate can be lower than Bondi-Hoyle by up to five orders of magnitude.

For example, in Srg A*, $\dot{M} \sim 10^{17}$ g s^{-1} , instead of 10^{22} g s^{-1}.

Accretion with non-zero angular momentum

Of course, the gas motions at large radii are never perfectly radial. Deviations can occur either because of thermal motion ($\sim$10 km s^{-1}), or motion associated with typical stellar velocities ($\sim$100 km s^{-1}).

$l = \lambda c r_S$ (for the captured gas, where $r_S = 2GM/c^2$)

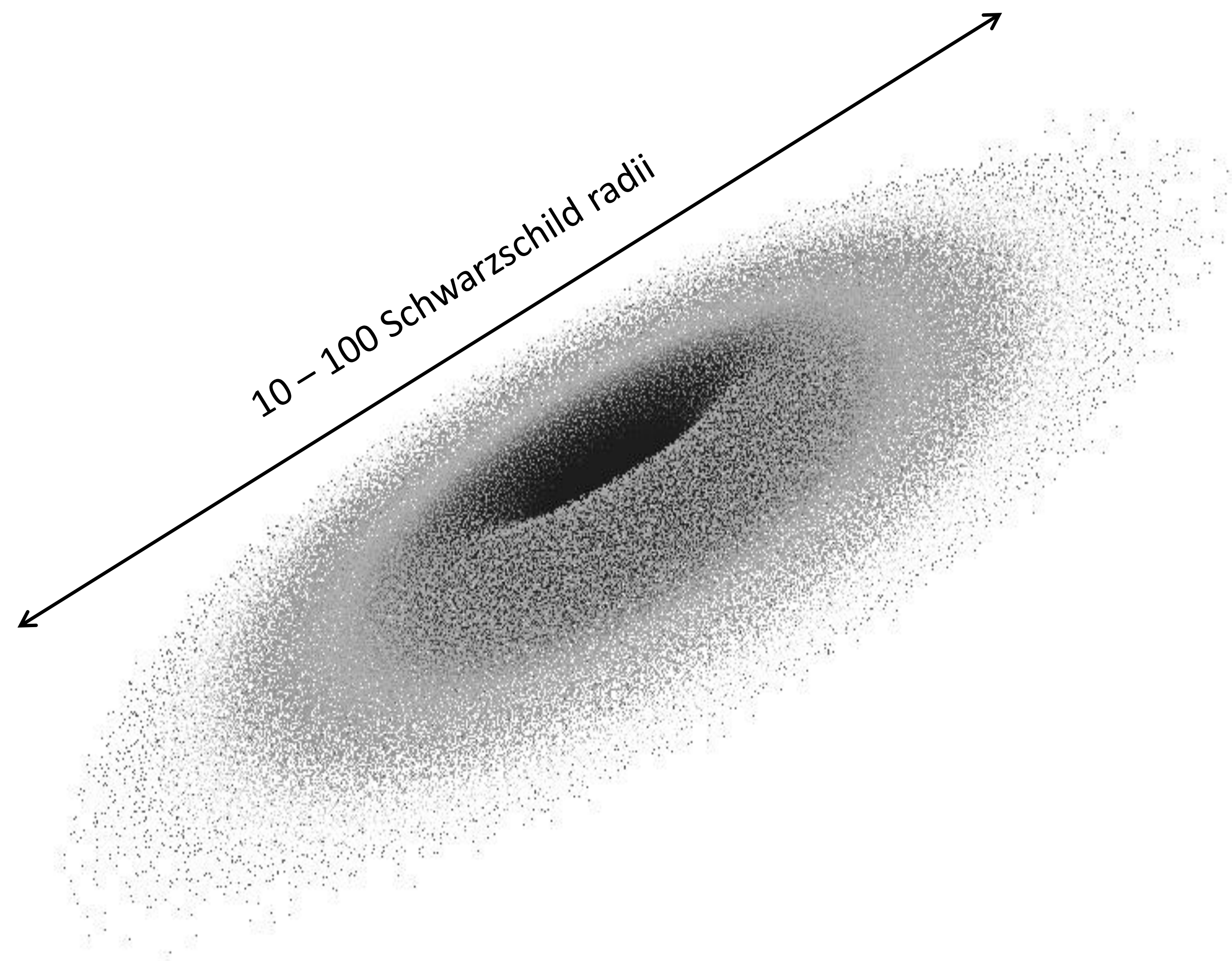
$\lambda c r_S = \sqrt{GMr}$ (for a Keplerian orbit)

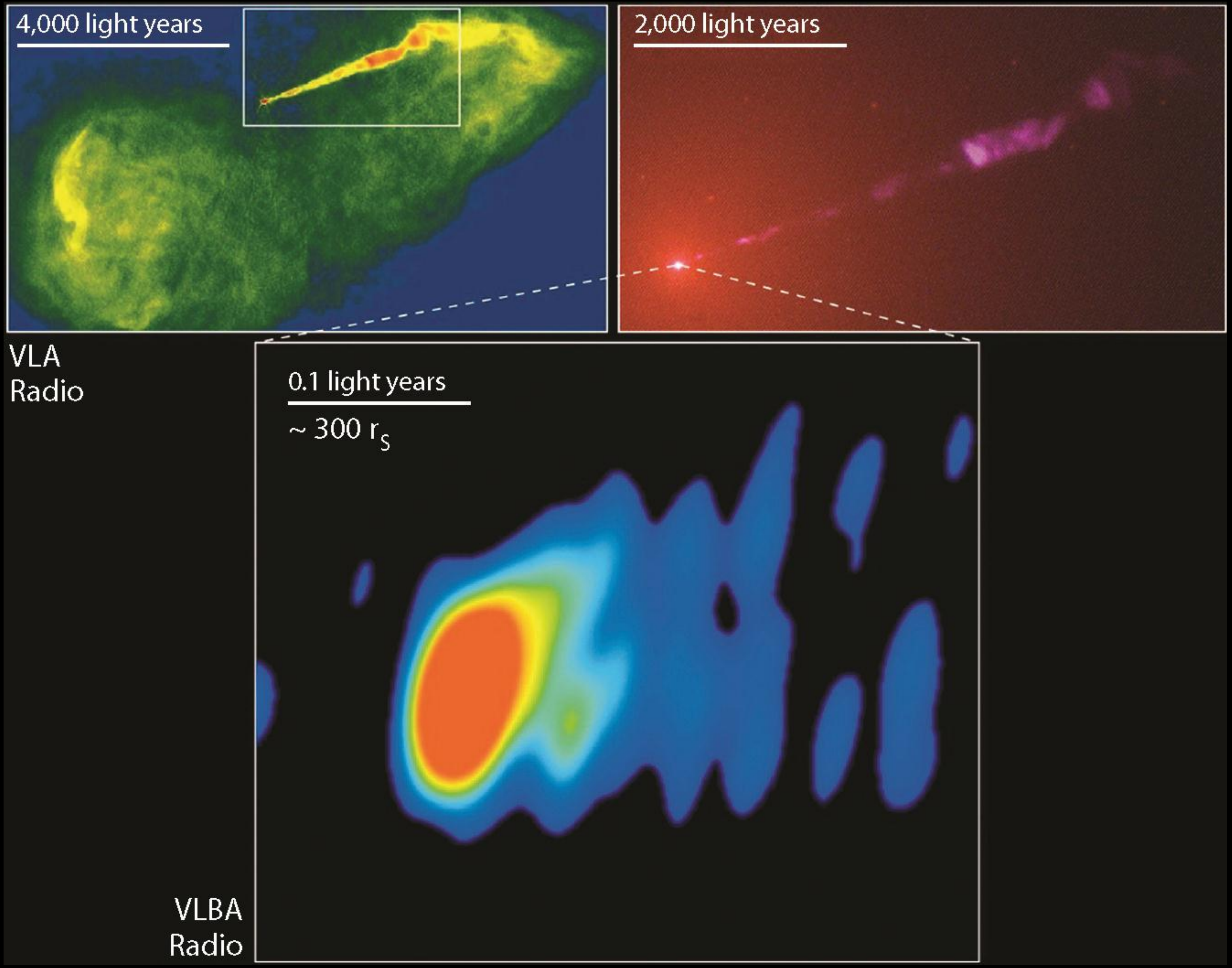
$$r_{circularization} = 2\lambda^2 r_S$$

$$c_s 10^5\, r_S \sim \lambda c r_S \quad \rightarrow \quad \lambda \sim 3$$

$$(100\ km\, s^{-1})\, 10^5\, r_S \sim \lambda c r_S \quad \rightarrow \quad \lambda \sim 30$$

So the gas circularizes somewhere between 10 and 1,000 Schwarzschild radii away from the black hole. Viscosity then drives the accretion from there.

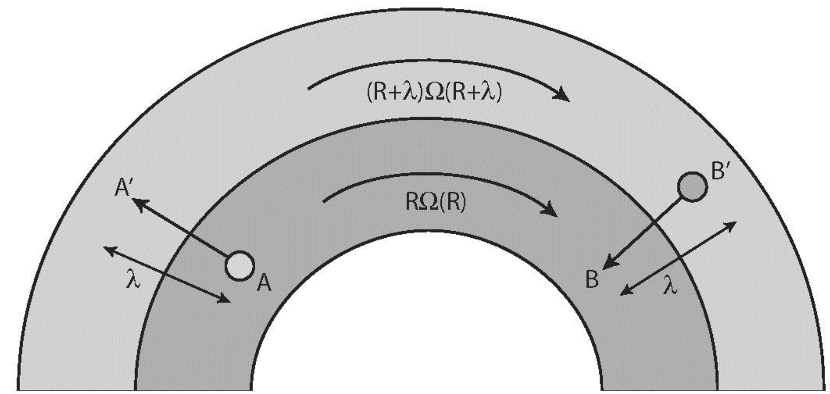

10 – 100 Schwarzschild radii



4,000 light years
2,000 light years
VLA
Radio
0.1 light years
~ 300 r_S
VLBA
Radio

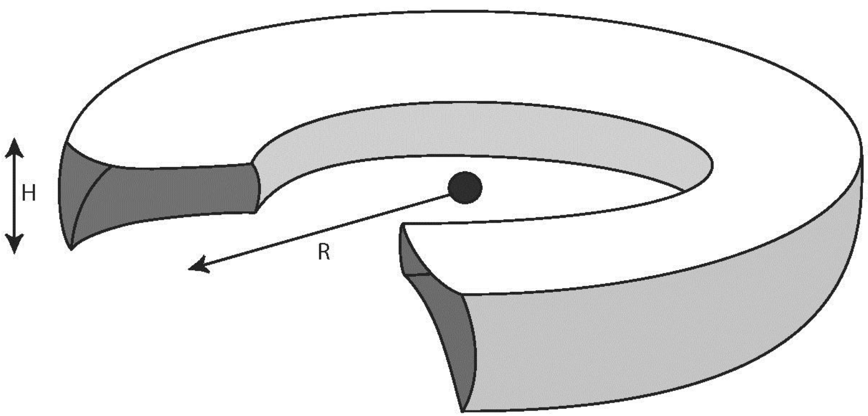
Disk Viscosity

$$\tau = \dot{M}_{A\to A'}(R+\lambda)R\Omega(R) -$$
$$\dot{M}_{B'\to B}R(R+\lambda)\Omega(R+\lambda)$$



In steady state

$$\dot{M}_{A\to A'} = \dot{M}_{B'\to B} = (2\pi RH)\rho(R)\tilde{v}$$



$$\tau = (2\pi RH)\rho(R)\tilde{v}R(R+\lambda)[\Omega(R) - \Omega(R+\lambda)]$$

$$\tau = -(2\pi RH)\rho(R)\tilde{v}R(R+\lambda)\lambda\Omega'(R)$$

$$\boxed{\tau = -2\pi\nu\rho HR^3\Omega'(R)}$$

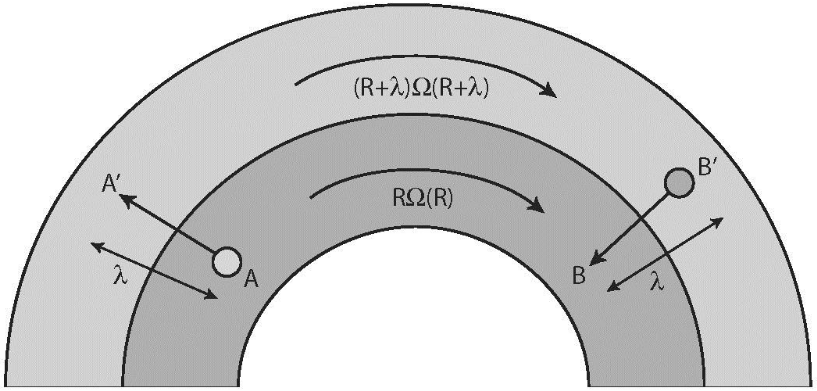
(coefficient of kinematic viscosity) $\boxed{\nu \equiv \tilde{v}\lambda}$

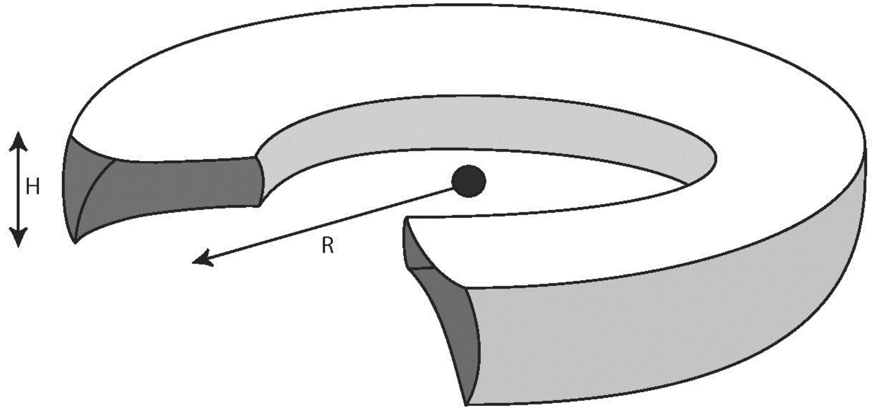
$(\alpha{\sim}1)$ $\boxed{\nu \equiv \alpha c_s H}$

Disk Viscosity

$$\tau \sim (2\pi HR)\rho v_R R^2 \Omega$$

$$\boxed{v_R \sim \nu \frac{\Omega\prime}{\Omega} \sim \frac{\nu}{R}}$$





$$\boxed{\tau = -2\pi\nu\rho H R^3 \Omega'(R)}$$

$$\boxed{\nu \equiv \tilde{v}\lambda}$$

$$\boxed{\nu \equiv \alpha c_s H}$$

One Temperature vs Two

If the cooling rate cannot keep up with the heating rate due to dissipation, then heat builds up and is advected inwards.

$t_\phi \equiv r/v_\phi = {\Omega_K}^{-1}$ (dynamical time scale)

$t_{visc} \equiv r/v_r$ (viscous time scale)

$t_z \equiv H/c_s$ (hydrodynamic time scale)

$t_{th} \equiv (\text{heat content})/(\text{dissipation rate})$ (thermal time scale)

$$v_r \sim \nu/r \sim \alpha c_s H/r$$

$$t_{visc} \sim (r/H)\left(v_\phi/\alpha c_s\right)t_\phi \gg t_\phi$$

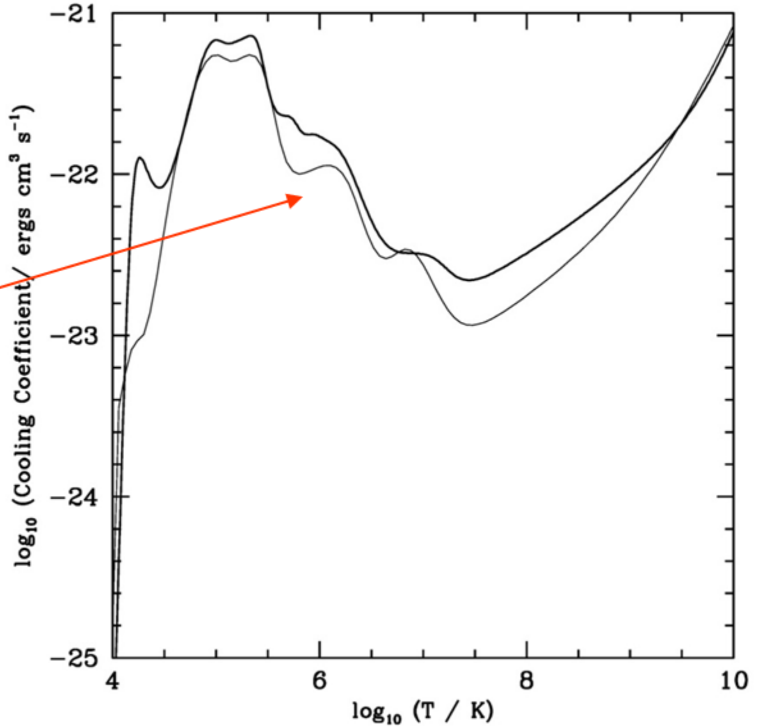
Similarly, $t_{th} \sim \left({c_s}^2/{v_\phi}^2\right) t_{visc} \ll t_{visc}$ (the ratio of momentum fluxes).

$$t_z \sim t_\phi$$

One Temperature vs Two

$$\boxed{t_\phi \sim t_z \ \leq t_{th} \ \ll \ t_{visc}}$$

Under some conditions, the cooling rate actually decreases as T increases, and this leads to an instability that drives the ions to a very high temperature on a thermal time scale, since the gas expands quickly.



The two-temperature possibility arises because most of the energy is carried by the ions, whereas most of the cooling is by the electrons. The temperature difference is matched to the dissipation rate:

$$\boxed{D(r) = (3/2)\nu_{coll}\, n_e k(T_i - T_e)}$$

One Temperature vs Two