Air Showers and Hadronic Interactions (1/3)



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Part I: Introduction to air shower physics

Particle interactions and cascades

Centaurus A



M31

Four-momentum kinematics

Calculation with four-momenta (c=1)
$$p = (E, \vec{p})$$

$$p_1 \cdot p_2 = E_1 E_2 - \vec{p_1} \cdot \vec{p_2}$$

$$p^2 = E^2 - \vec{p}^2 = m^2$$

Resonance production



Energy-momentum conservation

$$p_1 + p_2 = p_3$$

Mass of produced resonance

$$p_3^2 = m_3^2 = (p_1 + p_2)^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$
$$= p_1^2 + 2p_1 \cdot p_2 + p_2^2 = m_1^2 + m_2^2 + 2E_1E_2 - 2\vec{p}_1 \cdot \vec{p}_2$$

Laboratory and center-of-mass system

Mass of produced resonance, valid in any reference system

$$p_3^2 = m_3^2 = (p_1 + p_2)^2$$

Lab system:
$$p_1 = (E_1, \vec{p}_1)$$
 $p_2 = (m_2, \vec{0})$

$$m_3^2 = 2m_2E_1 + m_1^2 + m_2^2$$

Center-of-mass system (CMS, *): \vec{p}_1^{\star}

$$\vec{p}_1^\star = -\vec{p}_2^\star$$

$$m_3^2 = (E_1^{\star} + E_2^{\star})^2$$
 CMS energy: $E_{\rm cm} = \sqrt{s} = E_1^{\star} + E_2^{\star}$

Cosmic ray flux and CMS energy



Ultra-high energy: 10²⁰ eV

Need accelerator of size of Mercury's orbit to reach 10^{20} eV with current technology



Acceleration time for LHC: 815 years

Extensive air showers



Simulation of shower development (i)



Proton shower of low energy (knee region)

Simulation of shower development (ii)



Simulation of air shower tracks (i)



Simulation of air shower tracks (ii)



Particles of an iron shower



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Iron 10¹³ eV

24929 m

Particles of an proton shower



Proton 10¹³ eV

21336 m

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Particles of a gamma-ray shower



Gamma 10¹³ eV

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Time structure in shower front



J.Oehlschlaeger, R.Engel, FZKarlsruhe

Cross section and interaction rate



Atmosphere and interaction length

Altitude (km)	Vertical depth (g/cm ²)	Local density (10 ⁻³ g/cm ³)	Molière unit (m)	Electron Cherenkov threshold (MeV)	Cherenkov angle (°)
40	3	3.8×10^{-3}	2.4×10^{4}	386	0.076
30	11.8	1.8×10^{-2}	5.1 × 10 ³	176	0.17
20	55.8	8.8 × 10 ⁻²	1.0×10^{3}	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1,032	1.23	76	21	1.36

US standard atmosphere

Atmospheric depth

$$\int \rho_{\rm air} \, \mathrm{d}l = X$$

Interaction length

$$\lambda_{\text{int}} = \frac{\langle m_{\text{air}} \rangle}{\sigma_{\text{int}}} = \frac{24160 \,\text{mb g/cm}^2}{\sigma_{\text{int}}}$$

$$\frac{\mathrm{d}P}{\mathrm{d}X_1} = \frac{1}{\lambda_{\mathrm{int}}} e^{-X_1/\lambda_{\mathrm{int}}}$$

Typical values

$$\lambda_{\pi} \approx \lambda_{K} \approx 120 \,\mathrm{g/cm^{2}}$$

 $\lambda_{p} \approx 90 \,\mathrm{g/cm^{2}}$
 $\lambda_{Fe} \approx 5 \,\mathrm{g/cm^{2}}$

Hadronic cascades



Analytic cascade models of air showers

Electromagnetic showers: Heitler model



Electromagnetic showers: Cascade equations

Energy loss of electron:

$$\frac{\mathrm{d}E}{\mathrm{d}X} = -\alpha - \frac{E}{X_0}$$

Critical energy: $E_c = \alpha X_0 \sim 85 \,\mathrm{MeV}$ Radiation length: $X_0 \sim 36 \,\mathrm{g/cm^2}$

Cascade equations

$$\frac{\mathrm{d}\Phi_e(E)}{\mathrm{d}X} = -\frac{\sigma_e}{\langle m_{\mathrm{air}} \rangle} \Phi_e(E) + \int_E^{\infty} \frac{\sigma_e}{\langle m_{\mathrm{air}} \rangle} \Phi_e(\tilde{E}) P_{e \to e}(\tilde{E}, E) \mathrm{d}\tilde{E} + \int_E^{\infty} \frac{\sigma_{\gamma}}{\langle m_{\mathrm{air}} \rangle} \Phi_{\gamma}(\tilde{E}) P_{\gamma \to e}(\tilde{E}, E) \mathrm{d}\tilde{E} + \alpha \frac{\partial \Phi_e(E)}{\partial E}$$

$$X_{\max} \approx X_0 \ln\left(\frac{E_0}{E_c}\right) \qquad \qquad N_{\max} \approx \frac{0.31}{\sqrt{\ln(E_0/E_c) - 0.33}} \frac{E_0}{E_c}$$

Mean longitudinal shower profile



Calculation with cascade Eqs.

Photons

- Pair production
- Compton scattering

Electrons

- Bremsstrahlung
- Moller scattering

Positrons

- Bremsstrahlung
- Bhabha scattering

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Energy spectra of secondary particles



Number of photons divergent

- Typical energy of electrons and positrons $E_c \sim 80 \text{ MeV}$
- Electron excess of 20 30%
- Pair production symmetric
- Excess of electrons in target

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Muon production in hadronic showers



Assumptions:

- cascade stops at $E_{part} = E_{dec}$
- each hadron produces one muon

Primary particle proton

 π^0 decay immediately

 Π^{\pm} initiate new cascades

$$N_{\mu} = \left(\frac{E_0}{E_{\text{dec}}}\right)^{\alpha}$$
$$\alpha = \frac{\ln n_{\text{ch}}}{\ln n_{\text{tot}}} \approx 0.82 \dots 0.95$$

Superposition model

Proton-induced shower

$$N_{\rm max} = E_0/E_c$$

$$X_{\rm max} \sim \lambda_{\rm eff} \ln(E_0)$$

$$N_{\mu} = \left(\frac{E_0}{E_{\rm dec}}\right)^{\alpha} \qquad \alpha \approx 0.9$$

Assumption: nucleus of mass A and energy E_0 corresponds to A nucleons (protons) of energy $E_n = E_0/A$

$$N_{\rm max}^A = A\left(\frac{E_0}{AE_c}\right) = N_{\rm max}$$

$$X_{\text{max}}^{A} \sim \lambda_{\text{eff}} \ln(E_0/A)$$
$$N_{\mu}^{A} = A \left(\frac{E_0}{AE_{\text{dec}}}\right)^{\alpha} = A^{1-\alpha} N_{\mu}$$

Superposition model: correct prediction of mean Xmax

iron nucleus





Glauber approximation (unitarity)

$$n_{\text{part}} = \frac{\sigma_{\text{Fe}-\text{air}}}{\sigma_{\text{p}-\text{air}}}$$

Superposition and semi-superposition models applicable to inclusive (averaged) observables

Measured components of air showers



core distance (km)

Longitudinal shower profile



Mean depth of shower maximum



⁽RE, Pierog, Heck, ARNPS 2011)

Energy and composition measurement (Ne-Nµ)



Electromagnetic energy and energy transfer



Muon multiplicity correlation with missing energy



(RE, Pierog, Heck, ARNPS 2011)

Model dependence of correction to obtain total energy small

Different slopes for em. and hadronic showers

⁽RE, Pierog, Heck, ARNPS 2011)

Derivation of elongation rate theorem

Elongation rate theorem

$$D_e^{\text{had}} = X_0(1 - B_n - B_\lambda)$$

(Linsley, Watson PRL46, 1981)

$$B_n = \frac{d\ln n_{\rm tot}}{d\ln E}$$

Large if multiplicity of high energy particles rises very fast, **zero in case of scaling**

$$B_{\lambda} = -\frac{1}{X_0} \frac{d\lambda_{\text{int}}}{d\ln E}$$

Large if cross section rises rapidly with energy

Note: $D_{10} = \log(10)D_e$

Summary: Introduction to air showers

Air showers are easy to understand

- Heitler model of em. shower
- Matthews-Heitler model of muon production
- Superposition model (which is better than expected)
- Elongation rate theorem

Still to come

- Shower-to-shower fluctuations
- Lateral distribution of particles
- Shower age, universality features
- Model predictions and data interpretation
- Muon discrepancy (help!)

Part 2: Modeling hadronic interactions

Modeling of hadronic interactions

Time-of-flight walls

Typical particle multiplicities: 5 to 15 secondaries

Compilation of total cross sections

Simulation concepts: energy ranges

Minijet region (scaling violation)

???

Particle production in resonance region

Photoproduction of resonances

In proton rest frame:

 $E_{\gamma,\text{lab}} \approx 300 \text{ MeV}$

Decay branching ratio proton:neutron = 2:1 Mean proton energy loss 20% Decay isotropic up to spin effects

Well-established resonances in photoproduction

Baryon resonances and their physical parameters implemented in SOPHIA (see text). Superscripts ⁺ and ⁰ in the parameters refer to $p\gamma$ and $n\gamma$ excitations, respectively. The maximum cross section, $\sigma_{\text{max}} = 4m_{\text{N}}^2 M^2 \sigma_0 / (M^2 - m_{\text{N}}^2)^2$, is also given for reference

Resonance	М	Г	$10^{3}b_{\gamma}^{+}$	σ_0^+	$\sigma^+_{ m max}$	$10^3 b_{\gamma}^0$	σ_0^0	$\sigma_{ m max}^0$
Δ(1232)	1.231	0.11	5.6	31.125	411.988	6.1	33.809	452.226
N(1440)	1.440	0.35	0.5	1.389	7.124	0.3	0.831	4.292
N(1520)	1.515	0.11	4.6	25.567	103.240	4.0	22.170	90.082
N(1535)	1.525	0.10	2.5	6.948	27.244	2.5	6.928	27.334
N(1650)	1.675	0.16	1.0	2.779	7.408	0.0	0.000	0.000
N(1675)	1.675	0.15	0.0	0.000	0.000	0.2	1.663	4.457
N(1680)	1.680	0.125	2.1	17.508	46.143	0.0	0.000	0.000
$\Delta(1700)$	1.690	0.29	2.0	11.116	28.644	2.0	11.085	28.714
$\Delta(1905)$	1.895	0.35	0.2	1.667	2.869	0.2	1.663	2.875
Δ(1950)	1.950	0.30	1.0	11.116	17.433	1.0	11.085	17.462

Breit-Wigner resonance cross section

$$\sigma_{\rm bw}(s; M, \Gamma, J) = \frac{s}{(s - m_{\rm N}^2)^2} \frac{4\pi b_{\gamma} (2J + 1) s \Gamma^2}{(s - M^2)^2 + s \Gamma^2}$$

Direct pion production

Possible interpretation: p fluctuates from time to time to n and π^+

Lifetime of fluctuations

Length scale (duration) of hadronic interaction $\Delta t_{
m int} < 1 {
m fm} pprox 5 {
m GeV}^{-1}$

$$\Delta t \approx \frac{1}{\Delta E} = \frac{1}{\sqrt{k^2 + m_V^2 - k}} = \frac{1}{k(\sqrt{1 + m_V^2/k^2} - 1)} \approx \frac{2k}{m_V^2}$$

Fluctuation long-lived for k > 3 GeV

$$\Delta t \approx \frac{2k}{m_V^2} > \Delta t_{\rm int}$$

Multiparticle production: vector meson dominance

Photon is considered as superposition of ``bare'' photon and hadronic fluctuation

$$|\gamma\rangle = |\gamma_{\text{bare}}\rangle + P_{\text{had}}\sum_{i}|V_{i}\rangle$$

$$P_{\rm had} \approx \frac{1}{300} \ \dots \ \frac{1}{250}$$

Cross section for hadronic interaction $\sim 1/300$ smaller than for pi-p interactions

Putting all together: description of total cross section

- PDG: 9 resonances, decay channels, angular distributions
- Regge parametrization at higher energy
- Direct contribution: fit to difference to data

Many measurements available, still approximations necessary

SOPHIA (Mücke et al. CPC124, 2000)

Comparison with measured partial cross sections

Comparison with measured partial cross sections

Measurement of nucleus disintegration

Effective em. dissociation cross section

Example: photo-dissociation of nuclei

Example: Greisen-Zatsepin-Kuzmin (GZK) effect

Energy loss times interaction probability

Radiation fields as possible target

Calculation of energy loss length

$$x_{\text{loss}}(E) = \frac{E}{dE/dx} = \frac{\lambda(E)}{\kappa(E)} \qquad \qquad \text{interaction length}$$

inelasticity $\kappa(E) = \frac{\langle \Delta E \rangle}{E}$

$$\frac{1}{\lambda_{\rm ph}(E)} = \frac{1}{8E^2\beta} \int_{\epsilon_{\rm th}}^{\infty} d\epsilon \frac{n(\epsilon)}{\epsilon^2} \int_{s_{\rm min}}^{s_{\rm max}(\epsilon,E)} ds (s - m_p^2 c^4) \sigma_{p\gamma}(s)$$

$$s_{\min} = (m_p c^2 + m_{\pi^0} c^2)^2$$
 $s_{\max}(\epsilon, E) = m_p^2 c^4 + 2E\epsilon(1+\beta)$

$$\epsilon_{\rm th} = \frac{s_{\rm min} - m_p^2 c^4}{2E(1+\beta)}, \quad \beta^2 = 1 - \frac{m_p^2 c^4}{E^2}$$

Scaling with redshift:

$$x_{\text{loss}}(E,z) = (1+z)^{-3} x_{\text{loss}}[(1+z)E, z=0]$$

Energy loss length of nucleons

(Stanev et al., Phys. Rev. D62 (2000) 093005)

Mean loss rate vs. full simulation

Hadronic energy loss: stochastic process

(Achterberg 1999, Stanev et al., PRD62, 2000)

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Photodisintegration of nuclei

(Khan et al., Astropart. Phys. 23, 2005)

Example of nucleus disintegration path

⁽Yamamoto et al., APP20, 2004)

Energy considerations for nuclei

Energy of nucleus needed for formation of giant dipole resonance in CMB

Nucleus at rest

Nucleus with E_A in CMB field

13 MeV

$$s = (p_{\gamma} + p_A)^2$$

= $p_{\gamma}^2 + p_A^2 + 2(p_{\gamma} \cdot p_A)^2$
= $(Am_p)^2 + 2Am_p E_{\gamma}$

 $s = (Am_p)^2 + 2E_{\gamma}^{\text{CMB}}E_A(1 - \cos\theta)$

$$E_{\gamma}^{\text{CMB}} \ge A \frac{m_p E_{\gamma}}{(1 - \cos \theta) E_A}$$

Iron: $E_A \sim 3 \ 10^{20} \text{ eV}$ Helium: $E_A \sim 2 \ 10^{19} \text{ eV}$

Light nuclei disintegrate very fast while traveling through CMB

Comparison of energy loss lengths

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Summary: Modeling of resonance region

No fundamental problem in resonance region

- Large amount of data exists (still not perfect)
- Careful implementation needed
- Several simulation codes available

Application to GZK processes

- Processes reasonably well understood
- Remarkable coincidence of energy thresholds
- Light nuclei disintegrate very fast
- Largest uncertainties coming from IR and UR background fields

Energy loss length of photons

(Fodor, Katz & Ringwald, 2002)

E[eV]